Interference and Diffraction

Introduction

In this lab you will investigate two important phenomena in physical optics: the interference and diffraction of light waves. These effects allow you control the amplitude and direction of propagating light; for example, the quality of telescope performance can be limited by the mirror size. The resulting blurriness from diffraction through a finite aperture degrades the resolution and can prevent a sufficiently sharp image from being formed.

Light is an electromagnetic wave which is generally composed of fields of many different wavelengths (i.e., it is polychromatic, consisting of multiple colors), and it would seem difficult to analyze the net effect of all of these different wavelengths. However, the effect can be understood by considering only what happens for a monochromatic wave (one with a single wavelength) then adding the fields of all of the colors present. Thus, by analysis of the monochromatic case, we can in principle understand what will happen in the polychromatic case. We will deal with the monochromatic case in this lab.

An electromagnetic (light) wave can be pictured as a combination of electric ($\vec{E}$) and magnetic ($\vec{H}$) fields whose directions are perpendicular to the propagation direction of the wave (see Figure 1). One way of thinking about light fields is to use the concept of a wavefront. If we take a snapshot in time of the electric field along the propagation axis, there are places on the wave where the field is a maximum, zero, and a maximum in the other direction. These represent different phases of the wave (e.g., crests and troughs). The phase changes continuously along the propagation direction.

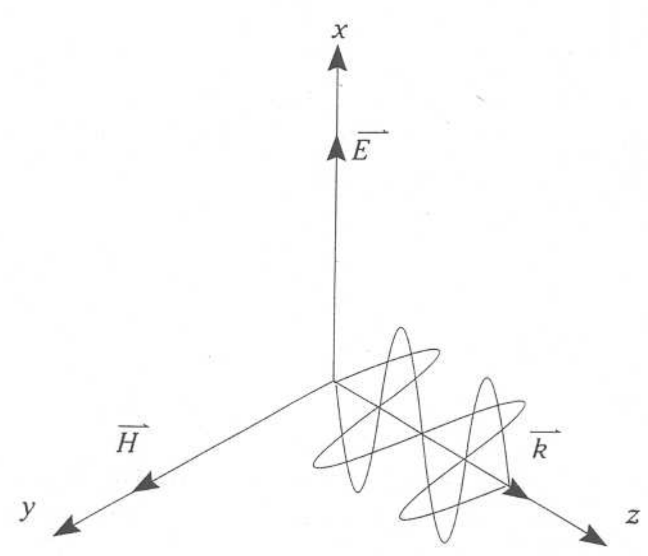

Figure 1: *Schematic of a monochromatic electromagnetic wave propagating along the $\hat{z}$ axis.*

Let's consider where the places where electric field amplitude is a maximum (the crests). In three dimensions, the points which have this same amplitude form a surface of constant phase, or wavefront.

One of the simplest examples is a plane wave, in which the light field is made up of plane surfaces of constant phase which are perpendicular to the propagation direction. The surface of constant phase propagates as the wave propagates. A spherical wavefront is formed by a point source, a fictitious source of infinitely small dimensions, which emits isotropically in all directions (Figure 2). The wavefront travels outward in all directions producing wavefronts consisting of spherical shells centered at the source. These are spherical waves. In the limit where you get very far from the point source, the radius of the spherical wavefront is so large that the curvature approaches zero and the wavefronts approximate plane waves. Spherical waves can also be formed by focusing plane waves to a point with a lens (and vice versa: a spherical wave can be focused into a plane wave by a lens in the reverse process).

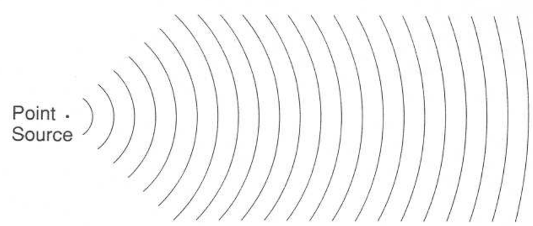

Figure 2: *Spherical wavefronts propagating from a point source. Far from the source the radius of the wavefront is large, and the wavefronts approximate plane waves.*

This is how single wavefronts behave, but what happens if two wavefronts are present in the same region of space? In this case, we apply the principle of superposition. Where waves overlap in the same region of space, the resulting field at that point in space and time is found by adding the electric fields of the individual waves at a point. This is a vector sum since the directions of the fields need not be the same.

Neither our eyes nor any light detector (sensor) can see the electric field of a light wave. Instead, the square of the time-averaged field is measured at each point on the detector. This is the irradiance of light, which, when a constant of proportionality is included, has units of Watts per square meter (W m^{-2}).

In order to predict the behavior of the wavefront as it progresses, we can invoke Huygens's Principle. Given an wavefront of arbitrary shape, we can locate an array of point sources on the wavefront, so that the strength of each point source is proportional to the amplitude of the wave at that point. Allow these point sources to propagate for a time t, so that the radii of the individual wavefronts are equal to ct and then add the wavefronts (see Figure 3). The resulting envelope is the net wavefront at time t after the initial wavefront.

Diffraction of light arises from the effects of apertures and interface boundaries as the light propagates. When a plane wave illuminates a slit, the resulting wave pattern that passes the slit can be constructed using Huygens's Principle by representing the wavefront as a collection of point sources all emitting in phase. The form of the irradiance pattern that is observed depends on the distance from the diffraction aperture, the size of the aperture, and the wavelength of the light.

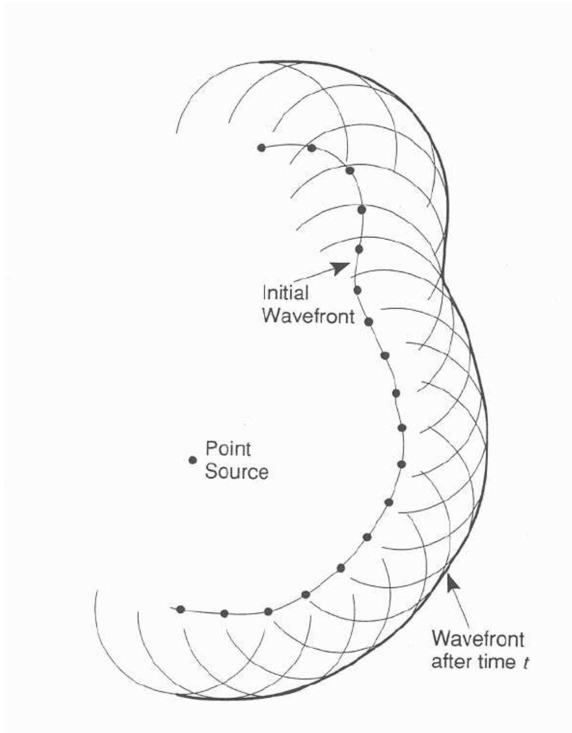

Figure 3: *Huygens's construction of a propagating wavefront of arbitrary shape.*

If the diffracted light is examined close to the aperture, the pattern will resemble the aperture with a few surprising variations (such as finding a point of light in the shadow of a circular mask). This is known as Fresnel diffraction and is somewhat difficult to calculate.

Far from the aperture, the pattern changes into a Fraunhofer diffraction pattern, which is easier to calculate and, in most cases, determines the optical limitations of most precision optical systems. For a large aperture, the blurring from diffraction is reduced and the image formed will be sharper (the resolution is increased). The simplest pattern formed is that due to a long slit aperture. Since the length of the slit is much larger than its width, the strongest diffractive effect is that due to the narrower width. The resulting pattern on a distant screen contains maxima and minima (fringes, see Figure 4a). The light is diffracted in a direction perpendicular to the slit edges. At distances far from the slit, the Fraunhofer diffraction pattern does not change in shape, but only in size (i.e., the angular separation of components does not change).

For circular apertures, the diffraction pattern is also circular (see Figure 4b) and is made up of a series of concentric light and dark rings. The region within the first dark ring is called the Airy disk, and the angular separation θ between the central maximum of the Airy disk and the first dark ring is given by:

$$\sin\theta = 1.22\lambda/D \tag{1}$$

where D is the aperture diameter. For large D (i.e., $D \gg \lambda$), the angular separation is small enough that $\sin\theta$ can be replaced with θ, the angular half-width of the Airy disk. The measurement of the diameter D of different size pinholes using the observed Airy disk pattern is part of this lab.

One good approximation of a point source is a bright star. A pair of stars close to one another can give a measure of the diffraction limits of a system which is called the resolution. The resolution of

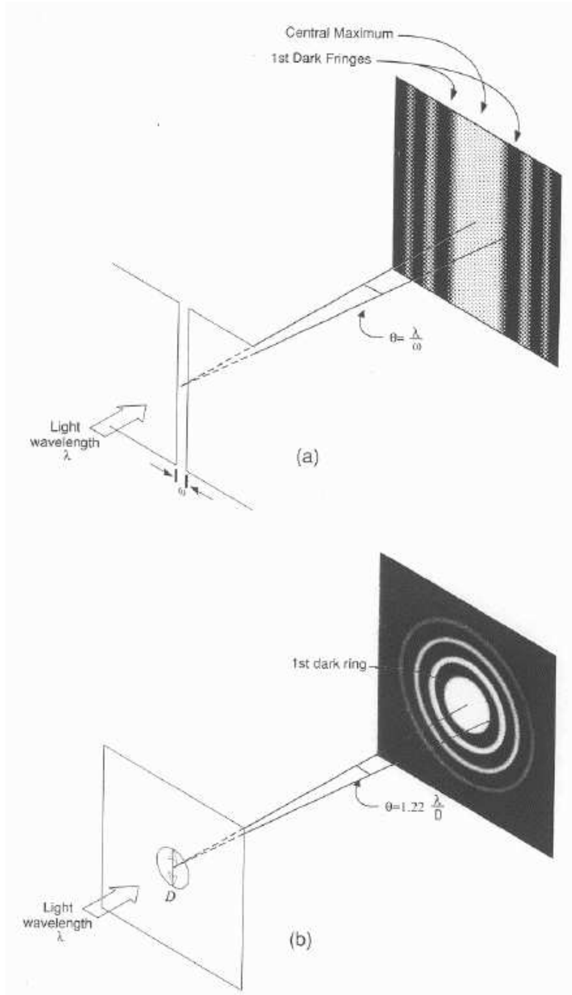

Figure 4: *Diffraction of light by apertures: (a) single slit, (b) circular aperture.*

a system can be determined by the smallest angular separation between such sources that would still allow them to be distinguished. A limit of the resolution that has been used in many instances is that two point sources are considered just resolvable if the maximum of the diffraction pattern of one point source falls on the first dark ring of the pattern of the second point source (Figure 5). This condition for the resolution θ_R

$$\theta_R = 1.22\lambda/D \tag{2}$$

is called the Rayleigh criterion.

Experiment

In this experiment you will measure the diffraction effects of circular apertures and will quantitatively determine the aperture sizes using the diffraction pattern. The diffraction associated with the size of the aperture determines the resolving power of all optical instruments, from the electron microscope to giant radio telescope dishes used in astronomy. In addition, you will discover that a solid object not only casts a shadow, but it is possible for a bright spot to appear in the center of the shadow!

Warning: This experiment will be performed in a darkened room. Extreme care should be taken when using the laser. Your pupils will be expanded and will let in 60 times more

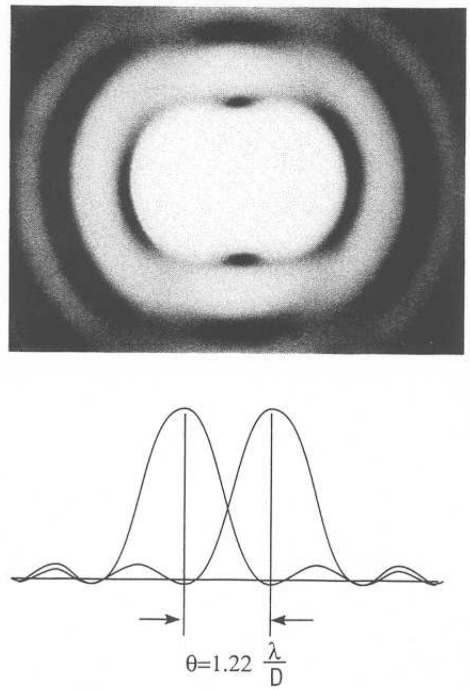

Figure 5: *The Rayleigh criterion. The plot of intensity along the a line between the centers of the two diffraction patterns is shown below a photo of two sources barely resolved a s specified by the Rayleigh criterion.*

light than a lighted room. Do not look at direct specular reflections or the direct laser beam. Please wear the appropriate eye protection when the laser is in use.

Using the experimental setup described below and shown in Figure 6, you will shine a laser beam through circular apertures. The diffraction pattern behind the aperture in each case will have concentric light and dark rings which you will measure to derive the aperture diameter. The experimental setup is shown in Figure 6.

1. Mount a laser assembly (item A) at the rear of the optical table. Tape an index card with a small (2 mm) hole in it to the front of the laser, so that the laser beam can pass through it. This card will be used to monitor the reflections from the optical components as they are inserted into the beam.

2. Mount a mirror steering assembly (item B) about 4 inches from the edge of the optical table. Adjust the height until the laser beam hits the center of the mirror. Then rotate the post until the laser beam is reflected at a right angle and travels parallel to the optical table edge.

3. Place a second mirror steering assembly (item C) in the path of the beam about 4 inches from the optical table edge. Adjust and rotate the mirror until the laser is reflected at a right angle parallel to the optical table edge. The beam should be pointing back in the direction from whence it originally came (anti-parallel to the original beam coming from the laser).

4. Place an index card in a target hold assembly (item D) and set it at the end of the optical table in the path of the beam so that the beam hits the center of the card. This will be the target screen.

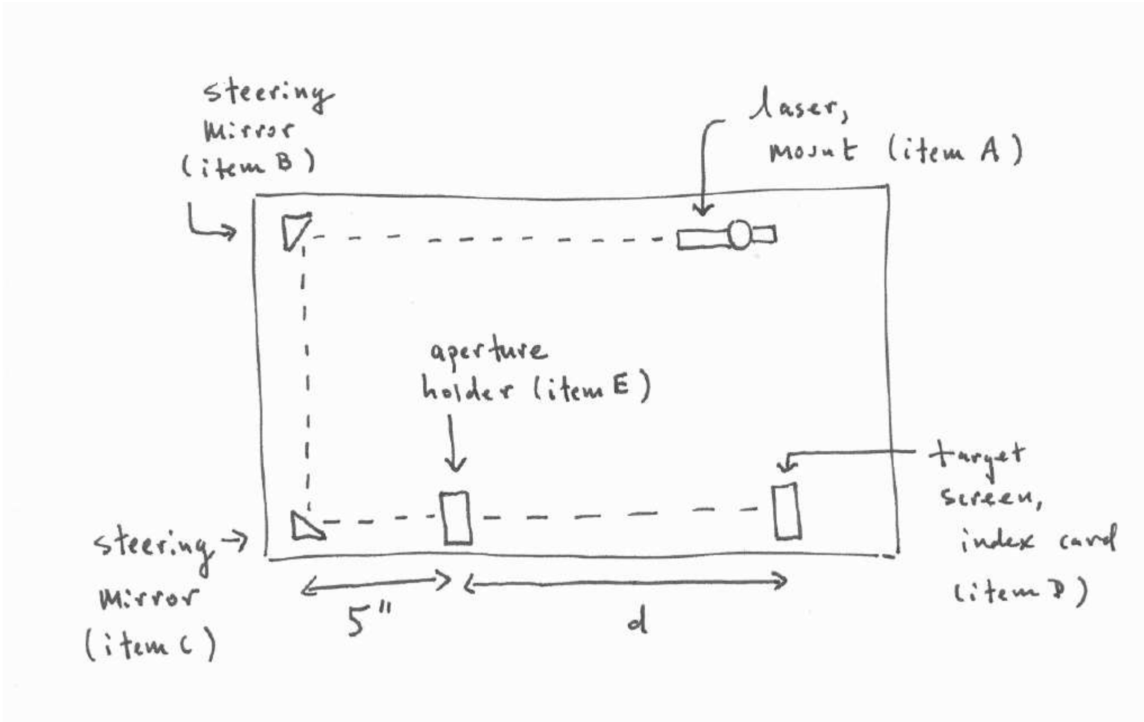

Figure 6: *Schematic of the experimental setup.*

5. Mount a lens chuck assembly (item E) in the path of the beam five inches after the second mirror assembly (item C) (i.e., the beam will hit item E after item C). This will be the aperture holder.

6. Place the first pinhole target P1 into the lens chuck assembly (item E). Adjust it so that the beam strikes the target at the center. The pinhole target will reflect a large percentage of the beam.

7. Adjust the second mirror steering assembly (item C) so that the laser beam fills the pinhole (item E). Look for a bright red glow from the back side of the pinhole target.

8. Carefully adjust the second mirror steering assembly (item C) to produce the brightest image on the target screen (item D). You should see a bright central circle surrounded by dark and light circular bands. This is the Airy disk pattern.

9. Measure the distance d from the pinhole target (item E) to the target screen (item D). Mark and measure the diameter of the first dark circular band around the bright central circle in the diffraction image. This is a measure of the amount of diffraction caused by the pinhole.

10. Since the angular diffraction is small (since $D \gg \lambda$), the sine and tangent of the diffraction angle are about equal:

$$\tan\theta \approx \sin\theta = 1.22\lambda/D \tag{3}$$

The tangent of found by dividing the radius (be careful: not the diameter!) of the first dark band by the distance d from the pinhole to the target screen. Calculate the diameter D of the pinhole using the wavelength of the He-Ne laser, $\lambda = 633$ nm.

Note: Be sure to include the uncertainties in each of your measurements and derive an uncertainty in your final derived value for the pinhole diameter.

11. Replace the pinhole target P1 with the next pinhole target P2. Repeat the necessary measurements and calculations to determine the pinhole diameter (again with uncertainty) for P2.

12. Assemble a 6:1 beam expander between the first and second mirror steering assemblies (between items B and C). Replace the pinhole target with the third pinhole target P3. The beam expander allows the laser beam to completely fill the larger aperture in this pinhole target. Since the pinhole diameter is large, the corresponding Airy disk size θ will be small. Therefore we need a large distance d to the screen in order to have the Airy disk diameter be measurable. Replace the target screen (item D) with another mirror steering assembly which reflects the laser beam toward a wall which is more than 10 feet away. Using the same measurements and methods as before, compute the pinhole diameter (with uncertainty).

13. Now replace the pinhole target P3 with the Fresnel target. Look at the diffraction pattern on the target screen. Note that the center of the image has several bright and dark rings. This is Fresnel diffraction. Depending on the distance between the Fresnel target and the target screen, the center of the pattern may be light or dark. Although the Fresnel target has a central obscuring circle, note that there is still light at the center of the pattern. The bright spot at the center is sometimes called the Poisson Spot or the Spot of Arago.

14. Now examine the shadows of other objects put in the expanded laser beam. Pencil points, wires, and small beads on a string are all good objects that give interesting Fresnel patterns. Note how the patterns change as you move the objects along the beam direction. Sketch some of the patterns.

Turn In

Please turn in explanations of your setup, measurements, and uncertainties. Include your derived pinhole diameters and a comparison of the real pinhole diameter values with their measured values. Make statements about their consistency. If the values were not consistent, what went wrong? What could account for the difference? What were the largest sources of uncertainty? etc. Please also include some sketches of your Fresnel patterns from other kinds of objects.